# Sequenced Units for the Common Core State Standards in Mathematics
# High School Algebra II

Algebra II extends the knowledge students have of algebraic and statistical concepts. They have investigated linear, exponential, and quadratic functions in previous years. Algebra II further develops important mathematical ideas introduced in Algebra I by extending techniques to solve equations and students' knowledge of functions by studying inverses and new function families: polynomial, radical, trigonometric, and rational functions. Students will also spend a significant portion of the school year studying probability and statistics. There are some (+) standards that are included in this course because the standards naturally support the assessed Algebra II content.

This document reflects our current thinking related to the intent of the Common Core State Standards for Mathematics (CCSSM) and assumes 160 days for instruction, divided among 11 units. The number of days suggested for each unit assumes 45-minute class periods and is included to convey how instructional time should be balanced across the year. The units are sequenced in a way that we believe best develops and connects the mathematical content described in the CCSSM; however, the order of the standards included in any unit does not imply a sequence of content within that unit. Some standards may be revisited several times during the course; others may be only partially addressed in different units, depending on the focus of the unit. Strikethroughs in the text of the standards are used in some cases in an attempt to convey that focus, and comments are included throughout the document to clarify and provide additional background for each unit.

Throughout Algebra II, students should continue to develop proficiency with the Common Core's eight Standards for Mathematical Practice:

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

These practices should become the natural way in which students come to understand and do mathematics. While, depending on the content to be understood or on the problem to be solved, any practice might be brought to bear, some practices may prove more useful than others. Opportunities for highlighting certain practices are indicated in different units in this document, but this highlighting should not be interpreted to mean that other practices should be neglected in those units.

When using this document to help in planning your district's instructional program, you will also need to refer to the CCSSM document, relevant progressions documents for the CCSSM, and the appropriate assessment consortium framework.

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| Unit 1: Arithmetic and geometric sequences | Suggested number of days: 10 |
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| Students' will connect their prior study of algebraic patterns with the concepts in this unit. Students explore the basic characteristics of arithmetic and geometric sequences and series, connecting these ideas to functions whose domains are a subset of the integers. They find explicit formulas, recursive processes, and sums. Students derive summation formulas for finite arithmetic and geometric series. Finally, they explore the notions of convergence and divergence as they develop the formula for the sum of an infinite geometric series. | |
| **Common Core State Standards for Mathematical Content** | **Comments** |
| **Seeing Structure in Expressions — A-SSE**<br>B. Write expressions in equivalent forms to solve problems<br>4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*★<br><br>**Interpreting Functions — F-IF**<br>A. Understand the concept of a function and use function notation<br>3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.<br><br>**Building Functions — F-BF**<br>A. Build a function that models a relationship between two quantities<br>1. Write a function that describes a relationship between two quantities.★<br>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.<br>2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★<br><br>**Linear, Quadratic, and Exponential Models**★**— F-LE**<br>A. Construct and compare linear, quadratic, and exponential models and solve problems<br>2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | Students will have been introduced to the explicit expressions for arithmetic and geometric sequences, recursive processes (**F-IF.A.3**, **F-BF.A.1a**, and **F-LE.A.2)** and made connections to linear and exponential functions in their Algebra I course. In this course students will explore the sum of a finite series (**A-SEE.B.4**) and writing recursive and explicit formulas (**F-BF.A.2**) for sequences. Although **A-SSE.B.4** does not specify the derivation of the formula for the sum of a finite arithmetic series, this derivation is important for preparing students for the derivation of the formula of the sum of a finite geometric series (**A-SSE.B.4)** so we include it here. |
| **Common Core State Standards for Mathematical Practice**<br><br>6. Attend to precision.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. | Encourage students to use precise vocabulary with discussions of the formulas they derive (**MP.6**). To write functions that generate arithmetic and geometric sequences, students must look for a common ratio and constant addendum (**MP.7**). In derivations for the explicit formulas, recursive formulas, and sums of finite series students will benefit from the practice of examining repeated calculations and look for shortcuts (**MP.8**). Specifically, students can derive the formula for the sum of a finite geometric series by noticing the way terms cancel out. |

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| Unit 2: Quadratic relations and equations | Suggested number of days: 10 |
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| This unit extends students' previous work with quadratic relations and equations. In the context of quadratics, students are introduced to the complex number system and complex solutions (**N-CN.A.1 and 2**). Students use finite differences to fit quadratic models to data. They also make connections among the general, vertex, and factored forms of a quadratic function, and they learn how to transform between forms to obtain needed information about the function. | |
| **Common Core State Standards for Mathematical Content** | **Comments** |
| **Quantities★— N-Q**<br>A. Reason quantitatively and use units to solve problems.<br>2. Define appropriate quantities for the purpose of descriptive modeling.<br><br>**The Complex Number System — N-CN**<br>A. Perform arithmetic operations with complex numbers.<br>1. Know there is a complex number *i* such that $i^2 = –1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.<br>2. Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.<br>C. Use complex numbers in polynomial identities and equations.<br>7. Solve quadratic equations with real coefficients that have complex solutions.<br><br>**Reasoning with Equations and Inequalities—A-REI**<br>B. Solve equations and inequalities in one variable<br>4. Solve quadratic equations in one variable.<br>b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.<br>C. Solve systems of equations<br>7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = –3x$ *and the circle* $x^2 + y^2 = 3$.<br><br>**Building Functions — F-BF**<br>A. Build a function that models a relationship between two quantities<br>1. Write a function that describes a relationship between two quantities.★<br>a. Determine an explicit expression, a recursive process, or steps for calculation from a context.<br>B. Build new functions from existing functions<br>3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* | Students explored transformations of quadratic functions in Algebra I (**F-BF.B.3**). In this course students connect this previous work to the concept of an even function as it relates to quadratics. Students extend their work with quadratic equations from Algebra I to solving quadratic equations with complex roots (**A-REI.B.4**). Students continue their study of real-world applications by fitting data to a quadratic function and building quadratic functions that model situations (**S-ID.B.6a** and (**F-BF.A.1**). In doing so, students will describe and define variables (**N-Q.A.2**).<br><br>Work with modeling and fitting quadratic data in contextual situations will help students demonstrate **MP.4**. As students are introduced to the complex numbers they will need to decontextualize in order to represent and operate on complex numbers (**MP.2**). As students derive the equation of a parabola using the focus and directrix, they will need to make use of geometric structures such as the distance formula (**MP.7**). |


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| **Expressing Geometric Properties with Equations — G-GPE**<br>A. Translate between the geometric description and the equation for a conic section<br>2. Derive the equation of a parabola given a focus and directrix.<br><br>**Interpreting Categorical and Quantitative Data — S★-ID**<br>B. Summarize, represent, and interpret data on two categorical and quantitative variables<br>6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.<br>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*<br><br>**Common Core State Standards for Mathematical Practice**<br>2. Reason abstractly and quantitatively.<br>4. Model with mathematics.<br>7. Look for and make use of structure. | |

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| Unit 3: Polynomial functions and equations | Suggested number of days: 20 |
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This unit builds on students' previous work with linear and quadratic functions to help students make sense of the behavior they see in the larger family of polynomial functions. The unit leads students to understand that polynomials form a system analogous to the integers; namely, they are closed under operations of addition, subtraction, and multiplication. Students learn how polynomials model some behavior with varying rates of change, and they see how the degree of the polynomial relates to the number of real zeros and the number of local extreme values of the polynomial function. Students then apply this knowledge to choose appropriate models for situations based on how quantities in this situation vary with emphasis on short term and end behavior. Understanding polynomial functions is useful for future work in calculus and for curve fitting in statistics and computer science.

## Common Core State Standards for Mathematical Content

**The Complex Number System — N-CN**

C. Use complex numbers in polynomial identities and equations.

9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

**Seeing Structure in Expressions — A-SSE**

A. Interpret the structure of expressions

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

**Arithmetic with Polynomials and Rational Expression —A-APR**

B. Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

C. Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.*

**Reasoning with Equations and Inequalities—A-REI**

D. Represent and solve equations and inequalities graphically

11. Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are ~~linear,~~ polynomial~~, rational, absolute value, exponential, and logarithmic~~ functions.★

## Comments

Building on students' Algebra I work with linear and quadratic functions, the work with **A-SSE.A.2**, **A-APR.B.3**, **A-REI.D.11, F-IF.B.4 and 6**, **F-IF.C.9, F-BF.B.3** in this unit focuses on polynomials of degree 3 or greater. Odd and even symmetries should be extended from the discussion in the previous unit.

**N-CN.C.9** is included because of the close relationship between the Fundamental Theorem of Algebra and the Remainder Theorem (**A-APR.B.2**). Additionally, knowing the upper bound for the number of zeros for a polynomial is helpful when identifying zeros (**A-APR.B.3**).

Students will investigate and build polynomial functions that precisely communicate different characteristics using technology (**MP.5**). With their work with factoring polynomials and proving polynomial identities, students will become proficient with making use of structure and repeated reasoning (**MP.7** and **MP.8**).

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| **Interpreting Functions — F-IF**<br>B. Interpret functions that arise in applications in terms of the context<br>4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★<br>6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★<br>C. Analyze functions using different representations<br>7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★<br>c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.<br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*<br><br>**Building Functions — F-BF**<br>A. Build a function that models a relationship between two quantities<br>1. Write a function that describes a relationship between two quantities.★<br>b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*<br>B. Build new functions from existing functions<br>3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*<br><br>**Common Core State Standards for Mathematical Practice**<br>5. Use appropriate tools strategically.<br>7. Look for and make use of structure.<br>8. Look for and express regularity in repeated reasoning. | |

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| Unit 4: Rational functions and equations | Suggested number of days: 15 |
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The study of rational functions of the form *f*(*x*) =*p*(*x*)/*q*(*x*), where *p*(*x*) and *q*(*x*) are polynomials, naturally builds from the previous unit. Students learn about the general characteristics and behavior of rational functions and apply their knowledge of transforming functions to create and understand graphs of rational functions. Students formulate rational equations that arise from rational functions. They learn strategies for identifying and applying the algebraic skills needed to solve these rational equations in a variety of situations.

## Common Core State Standards for Mathematical Content

**Seeing Structure in Expressions — A-SSE**

A. Interpret the structure of expressions

2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

**Arithmetic with Polynomials and Rational Expression —A-APR**

D. Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write *a*(*x*)/*b*(*x*) in the form *q*(*x*) + *r*(*x*)/*b*(*x*), where *a*(*x*), *b*(*x*), *q*(*x*), and *r*(*x*) are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system.

**Creating equations★— A-CED**

A. Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from ~~linear and quadratic functions, and~~ simple rational ~~and exponential~~ functions.*

**Reasoning with Equations and Inequalities—A-REI**

A. Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple rational ~~and radical~~ equations in one variable, and give examples showing how extraneous solutions may arise.

**Reasoning with Equations and Inequalities—A-REI**

D. Represent and solve equations and inequalities graphically

11. Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are ~~linear, polynomial,~~ rational~~, absolute value, exponential, and logarithmic~~ functions.★

## Comments

The standards to which students have been exposed in Algebra I, extended in this unit to the study of rational functions and equations, are **A.SSE.A.2, A-CED.A.1, A-REI.A.1, A-REI.D.11, and N-Q.A.2.**

The focus of **A-REI.A.2** in this unit is on rational equations; radical equations are addressed in unit 5.

Connections should be made in this unit between the rewriting of rational expressions (**A-APR.D.6**) and the Remainder Theorem (**A-APR.B.2**) from the previous unit.

**F-BF.A.1b** is extended to rational functions in this unit.

**F-IF.C.1b** is included here in order to support **A-REI.D.11**.

Students are making sense of problems and preserving in their solutions (**MP.1**) when defining and building of functions (**F-BF.A.1b** and **N-Q.A.2**). Students may use technology to verify removable discontinuities and asymptotes (**MP.5**) as they study the solutions and graphs of rational functions. Students will also practice looking for and making use of structure (**MP.7**) as they rewrite rational functions (**A-SSE.A.2**).

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| **Interpreting Functions — F-IF**<br>C. Analyze functions using different representations<br>7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★<br>d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.<br><br>**Building Functions — F-BF**<br>A. Build a function that models a relationship between two quantities<br>1. Write a function that describes a relationship between two quantities.★<br>b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*<br><br>B. Build new functions from existing functions<br>4. Find inverse functions.<br>a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.<br><br>**Common Core State Standards for Mathematical Practice**<br>1. Make sense of problems and persevere in solving them.<br>5. Use appropriate tools strategically.<br>7. Look for and make use of structure. | |

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| Unit 5: Radical functions and equations | Suggested number of days: 15 |
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Students explore transformations on the parent square root function to model data and they formulate equations arising from square root functions. They explore solutions for these equations using tables and graphs, and they learn how the inverse relationship between square root and quadratic functions facilitates solving these equations analytically. They also investigate the notion of extraneous roots.

## Common Core State Standards for Mathematical Content

**Quantities★ — N-Q**

A. Reason quantitatively and use units to solve problems.

2. Define appropriate quantities for the purpose of descriptive modeling.

**The Real Number System — N-RN**

A. Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**Reasoning with Equations and Inequalities—A-REI**

A. Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
2. Solve simple ~~rational and~~ radical equations in one variable, and give examples showing how extraneous solutions may arise.

**Interpreting Functions — F-IF**

B. Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

**Building Functions — F-BF**

B. Build new functions from existing functions

4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.

## Comments

The standards to which students have been exposed in Algebra I, extended in this unit to the study of radical functions and equations, are **A-REI.A.1**, **F-IF.B.4**, **F-BF.B.4a**, and **N-Q.A.2**.

The focus of **N-Q.A.2**, **A-REI.A.2,** and **F-BF.B.4a** in this unit is on radical equations.

Students build proficiency with **MP.2** and **MP.5** as they operate, solve, and evaluate the reasonableness of solutions. Students may critique the work of others by checking for extraneous roots and the context from which the data models (**MP.3**). Additionally, students may use graphing technology (**MP.5**) as they investigate key features of radical functions (**F-IF.B.4**).

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| Common Core State Standards for Mathematical Practice | |
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| 2. Reason abstractly and quantitatively.<br>3. Construct viable arguments and critique the reasoning of other.<br>5. Use appropriate tools strategically. | |

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| **Unit 6: Exponential and logarithmic functions and equations** | **Suggested number of days: 15** |
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| In this unit students strengthen their understanding of the inverse relationship while making connections between exponential and logarithmic functions. Students learn how to use exponential functions to model changes in the values of the dependent variable produced through repeated multiplication by a positive constant. Through fitting models to data, students solidify their understanding of the characteristics of an exponential function. Students then numerically and graphically investigate the transcendental number *e* and learn about its role in the compounding of interest. Students develop properties of logarithms and use these properties and to solve problems algebraically. Finally, students explore the effects of the parameters on the graphs of exponential and logarithmic functions. | |
| **Common Core State Standards for Mathematical Content** | **Comments** |
| **Quantities★— N-Q**<br>A. Reason quantitatively and use units to solve problems.<br>2. Define appropriate quantities for the purpose of descriptive modeling.<br><br>**Seeing Structure in Expressions — A-SSE**<br>B. Write expressions in equivalent forms to solve problems<br>3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★<br>c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* $1.15^t$ *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*<br><br>**Creating equations★— A-CED**<br>A. Create equations that describe numbers or relationships<br>1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*<br><br>**Reasoning with Equations and Inequalities—A-REI**<br>D. Represent and solve equations and inequalities graphically<br>11. Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are ~~linear, polynomial, rational, absolute value,~~ exponential, and logarithmic functions.★<br><br>**Interpreting Functions — F-IF**<br>B. Interpret functions that arise in applications in terms of the context<br>4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ | The standards to which students have been exposed in Algebra I and previously in this course (**A-SSE.3c, A-CED.A.1**, **A-REI.D.11**, **F-IF.B.4** and **6**, **F-IF.C.9**, **F-IF.C.9**, **F-BF.A.1a** and **3**, and **S-ID.B.6a)** are extended in this unit to the study of exponential and logarithmic functions and equations.<br><br>Students previously constructed exponential functions (**F-LE.A.2**) in Algebra I; now students will work with more complex multi-step tasks. And, although students interpreted the parameters of exponential functions in terms of a context in Algebra I (**F-LE.B.5),** now students will be expected to work with exponential functions with domains not limited to the integers.<br><br>**F-BF.B.5** is included here to support the other standards in this unit.<br><br>Students will investigate data that can be modeled with exponential and logarithmic functions (**MP.4**). When calculating logarithms and exponents, students must recognize the importance of maintaining the mantissa and significant digits (**MP.6**). Students will |

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| 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★<br><br>C. Analyze functions using different representations<br>7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★<br>e. Graph exponential and logarithmic functions, showing intercepts and end behavior~~, and trigonometric functions, showing period, midline, and amplitude~~.<br><br>8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.<br>b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*<br><br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*<br><br>**Building Functions — F-BF**<br>A. Build a function that models a relationship between two quantities<br>1. Write a function that describes a relationship between two quantities.★<br>a. Determine an explicit expression, ~~a recursive process,~~ or steps for calculation from a context.<br><br>3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*<br><br>B. Build new functions from existing functions<br>5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.<br><br>**Linear, Quadratic, and Exponential Models★— F-LE**<br>A. Construct and compare linear, quadratic, and exponential models and solve problems<br>2. Construct ~~linear and~~ exponential functions, including ~~arithmetic and~~ geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).<br><br>4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.<br><br>B. Interpret expressions for functions in terms of the situation they model<br>5. Interpret the parameters in a linear or exponential function in terms of a context. | understand how to flexibly utilize properties of logarithmic operations while operating on exponential and logarithmic equations (**MP.2**). |

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| **Interpreting Categorical and Quantitative Data — S★-ID**<br>B. Summarize, represent, and interpret data on two categorical and quantitative variables<br>6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.<br>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize ~~linear, quadratic, and~~ exponential models.*<br>**Common Core State Standards for Mathematical Practice**<br>2. Reason abstractly and quantitatively.<br>4. Model with mathematics.<br>6. Attend to precision. | |

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| Unit 7: Probability | Suggested number of days: 15 |
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| Students last formally studied probability in Grade 7, when they found probabilities of simple and compound events and designed and used simulations. This unit builds on these concepts, as well as fundamental counting principles and the notion of independence, to develop rules for probability and conditional probability. Units 7, 8, and 9 may be treated as a modular group of units. However, it is important that the standards are taught in a timely manner because the majority of the standards in the three units are major content for Algebra II. | |
| **Common Core State Standards for Mathematical Content** | **Comments** |
| **Conditional Probability and the Rules of Probability — S★-CP**<br>A. Understand independence and conditional probability and use them to interpret data<br>1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").<br>2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.<br>3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.<br>4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.<br>5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.<br>B. Use the rules of probability to compute probabilities of compound events in a uniform probability model<br>6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.<br>7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.<br>8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B\|A) = P(B)P(A\|B), and interpret the answer in terms of the model.<br>9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems. | **S-CP.B.8, S-CP.B.9, S-MD.B.6, and S-MD.B.7** are included here to support the other standards in this unit.<br><br>Students should be encouraged to persevere when problem solving in this unit. Multiple solutions are common and should be recognized. Students can often make sense of complex contextual probabilities by considering a simpler analogous probability situation (**MP.1**). As students work to identify events for which probabilities are to be determined and rules to apply, encourage students to verify and critique the thinking of their classmates (**MP.3**). Students have the opportunity to demonstrate proficiency with **MP.6** by paying close attention to precise use of new vocabulary and writing complete sentences describing probabilities. |

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| **Using Probability to Make Decisions — S★-MD**<br>B. Use probability to evaluate outcomes of decisions<br>6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).<br>7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).<br><br>**Common Core State Standards for Mathematical Practice**<br>1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning of others.<br>6. Attend to precision. | |

| **Unit 8: The design of statistical studies** | **Suggested number of days: 15** |
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| Drawing correct conclusions from data is highly dependent on how the data are collected. In particular, "cause and effect" conclusions can only arise from properly conducted experiments, in which the researcher actively imposes a treatment. In this unit students study design of experiments based on three fundamental principles: control of outside variables, randomization, and replication within the experiment. This unit is an introduction to these and other key issues in experimental design. | |
| **Common Core State Standards for Mathematical Content** | **Comments** |
| **Making Inferences and Justifying Conclusions — S★-IC**<br>A. Understand and evaluate random processes underlying statistical experiments<br>1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.<br>B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies<br>3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.<br>6. Evaluate reports based on data.<br><br>**Common Core State Standards for Mathematical Practice**<br>1. Make sense of problems and persevere in solving them.<br>3. Construct viable arguments and critique the reasoning of others.<br>5. Use appropriate tools strategically. | Students must follow a reasoning process in order to develop a sound statistical study (**MP.1**). Have students communicate, justify, and ask questions to improve the designs of statistical studies (**MP.3**). Students should consider the role and limitations of technology in randomization (**MP.5**). |

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| Unit 9: Gathering data, making inferences, and justifying conclusions | Suggested number of days: 15 |
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| Student learn to use probability, relative frequencies, and discrete distributions to develop a conceptual understanding of the normal distribution and use the distribution to estimate population proportions. | |

## Common Core State Standards for Mathematical Content

**Interpreting Categorical and Quantitative Data — S★-ID**

A. Summarize, represent, and interpret data on a single count or measurement variable

4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

**Making Inferences and Justifying Conclusions — S★-IC**

A. Understand and evaluate random processes underlying statistical experiments

2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies

4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

## Common Core State Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
5. Use appropriate tools strategically.

## Comments

Students will continue with the work from the previous unit of evaluating reports (**S-IC.B.6**).

Students continue to grow their technology proficiencies (**MP.5**) by performing simulations (**S-IC.B.4**) and finding the area beneath a curve using various tools (**S-ID.A.4**). Students will plan and evaluate simulations (**MP.1**). As students work with data to make inferences and justify conclusions they must construct critiques of the reasoning of others (**MP.3**).

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<table>
<tr><th>Unit 10: Trigonometric functions</th><th>Suggested number of days: 15</th></tr>
<tr><td colspan="2">Students are introduced to periodic functions and define three trigonometric functions: y = sin α, y = cos α and y = tan α. Students learn to transform these functions just as they have transformed other types of functions. The parameter that affects the period of these functions is explored. Radians are introduced in connection to circular functions and trigonometric functions. By the end of this unit, students will have a conceptual understanding of how these functions are generated and used to model various situations.</td></tr>
<tr><td>Common Core State Standards for Mathematical Content<br><br>Quantities★ — N-Q<br>A. Reason quantitatively and use units to solve problems.<br>2. Define appropriate quantities for the purpose of descriptive modeling.<br><br>Interpreting Functions — F-IF<br>B. Interpret functions that arise in applications in terms of the context<br>4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★<br>6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★<br>C. Analyze functions using different representations<br>7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★<br>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.<br>9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.<br><br>Building Functions — F-BF<br>B. Build new functions from existing functions<br>3. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</td><td>Comments<br><br>The work with N-Q.A.2, F-IF.B.4, F-IF.B.6, F-IF.C.7e F-IF.C.9, and F-BF.B.3 in this unit is extended to trigonometric functions.<br><br>In the extension of the trigonometric functions to the unit circle, proficient students must use repeated reasoning (MP.8). Students will model real world situations with trigonometric functions (MP.4). Use of trigonometric vocabulary, such as (amplitude, frequency, period, midline, degree, and radian) aid in communicating precisely (MP.6).</td></tr>
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| **Trigonometric Functions — F-TF**<br>A. Extend the domain of trigonometric functions using the unit circle<br>1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.<br>2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.<br>B. Model periodic phenomena with trigonometric functions<br>5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★<br><br>**Trigonometric Functions — F-TF**<br>C. Prove and apply trigonometric identities<br>8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.<br><br>**Common Core State Standards for Mathematical Practice**<br>4. Model with mathematics.<br>6. Attend to precision.<br>8. Look for and express regularity in repeated reasoning. | |

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| **Unit 11: Choosing a function model** | **Suggested number of days: 15** |
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| In this unit, students draw upon their knowledge of different parent functions, including linear, polynomial, power, exponential, logarithmic and trigonometric functions, to model a variety of situations. The modeling standards students will likely revisit in unit are **A-SSE.B.3c** and **4**, **F-IF.B.4** and **6**, the **F-LE** domain, and **F-TF.5.** Students compare the fit of function models to various data sets. They use regression and learn about correlation to determine the strength of a model. Students use the model to make predictions in the context of the situation given. Additionally, students use their understanding of the behavior of functions and modeling to solve optimization problems. Students analyze problem situations, model the situation with a function, and find local maximum or minimum values of that function using graphs and tables, and interpret their solutions in the context of the problem given. In solving these problems, students use geometric relationships, numerical relationships and combinations of functions in various real-world applications. | |
| **Common Core State Standards for Mathematical Content** | **Comments** |
| **Quantities★— N-Q**<br>A. Reason quantitatively and use units to solve problems.<br>2. Define appropriate quantities for the purpose of descriptive modeling.<br><br>**Reasoning with Equations and Inequalities—A-REI**<br>C. Solve systems of equations<br>6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | Students have been exposed to **N-Q.2**, **A-REI.C.6** and **S-ID.B.6a** in Algebra I or previously in this course in the context of studying particular functions. Now students will analyze situations and choose the type of function that best models the situation. |
| **Interpreting Categorical and Quantitative Data — S★-ID**<br>B. Summarize, represent, and interpret data on two categorical and quantitative variables<br>6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.<br>a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*<br><br>**Common Core State Standards for Mathematical Practice**<br>3. Construct viable arguments and critique the reasoning of others.<br>4. Make sense of problems and persevere in solving them.<br>5. Use appropriate tools strategically. | Students create and analyze various models (**MP.4**) using tools such as computer algebra systems, spreadsheets, and graphing technology (**MP.5**). Students must defend the appropriateness of their models and any conclusions they draw based on those models (**MP.3**). |

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